Basic Concepts

- Enclosures consist of two or more surfaces that envelop a region of space (typically gas-filled) and between which there is radiation transfer. Virtual, as well as real, surfaces may be introduced to form an enclosure.
- A nonparticipating medium within the enclosure neither emits, absorbs, nor scatters radiation and hence has no effect on radiation exchange between the surfaces.
- Each surface of the enclosure is assumed to be isothermal, opaque, diffuse and gray, and to be characterized by uniform radiosity and irradiation.
The View Factor (also Configuration or Shape Factor)

- The view factor, \( F_{ij} \), is a geometrical quantity corresponding to the fraction of the radiation leaving surface \( i \) that is intercepted by surface \( j \).
  \[ F_{ij} = \frac{q_{i \rightarrow j}}{A_i J_j} \]
- The view factor integral provides a general expression for \( F_{ij} \).
  Consider exchange between differential areas \( dA_i \) and \( dA_j \):
  \[ dq_{i \rightarrow j} = I_i \cos \theta_i dA_i d\omega_{i \rightarrow j} = J_i \frac{\cos \theta_i \cos \theta_j}{\pi R^2} dA_i dA_j \]
  \[ F_{ij} = \frac{1}{A_i} \int_{A_i} \int_{A_j} \frac{\cos \theta_i \cos \theta_j}{\pi R^2} dA_i dA_j \]

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View Factor Relations

- Reciprocity Relation. With \( F_{ij} = \frac{1}{A_i} \int_{A_i} \int_{A_j} \frac{\cos \theta_i \cos \theta_j}{\pi R^2} dA_i dA_j \)
  \[ A_i F_{ij} = A_j F_{ji} \]
- Summation Rule for Enclosures.
  \[ \sum_{j=1}^{N} F_{ij} = 1 \]

From Table 13-1:

\[ F_{ij} = 1 - \left[ 1 - \left( \frac{D}{s} \right)^2 \right]^{1/2} + \left( \frac{D}{s} \right) \tan^{-1} \left[ \left( \frac{s^2 - D^2}{D^2} \right) \right]^{1/2} \]

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Blackbody Radiation Exchange

For a blackbody, \( J_i = E_{bi} \).

Net radiative exchange between two surfaces that can be approximated as blackbodies → net rate at which radiation leaves surface \( i \) due to its interaction with \( j \)

or net rate at which surface \( j \) gains radiation due to its interaction with \( i \)

\[
q_{ij} = q_{i\rightarrow j} - q_{j\rightarrow i}
\]

\[
q_{ij} = A_i F_{ij} E_{bi} - A_j F_{ji} E_{bj}
\]

\[
q_{ij} = A_i F_{ij} \sigma (T_i^4 - T_j^4)
\]

Net radiation transfer from surface \( i \) due to exchange with all \( (N) \) surfaces of an enclosure:

\[
q_i = \sum_{j=1}^{N} A_i F_{ij} \sigma (T_i^4 - T_j^4)
\]

General Radiation Analysis for Exchange between the \( N \) Opaque, Diffuse, Gray Surfaces of an Enclosure

Alternative expressions for net radiative transfer from surface \( i \):

\[
q_i = A_i (J_i - G_i) \rightarrow \text{Fig. (b)} \quad (1)
\]

\[
q_i = A_i (E_i - \alpha_i G_i) \rightarrow \text{Fig. (c)} \quad (2)
\]

\[
q_i = \frac{E_{bi} - J_i}{(1 - \varepsilon_i) / \varepsilon_i A_i} \rightarrow \text{Fig. (d)} \quad (3)
\]

Suggests a surface radiative resistance of the form: \( (1 - \varepsilon_i) / \varepsilon_i A_i \)
Suggests a space or geometrical resistance of the form:

\[ q_i = \sum_{j=1}^{N} A_i F_{ij} \left( J_i - J_j \right) = \sum_{j=1}^{N} \frac{J_i - J_j}{\left( A_i F_{ij} \right) \epsilon_i} \]

(4)

- Equating Eqs. (3) and (4) corresponds to a radiation balance on surface \( i \):

\[ E_{hi} - J_i = \sum_{j=1}^{N} \frac{J_i - J_j}{\left( A_i F_{ij} \right) \epsilon_i} \]

(5)

which may be represented by a radiation network of the form

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Methodology of an Enclosure Analysis

1. Apply Eq. (4) to each surface for which the net radiation heat rate \( q_i \) is known.
2. Apply Eq. (5) to each of the remaining surfaces for which the temperature \( T_i \), and hence \( E_{hi} \), is known.
3. Evaluate all of the view factors appearing in the resulting equations.
4. Solve the system of \( N \) equations for the unknown radiosities, \( J_1, J_2, \ldots, J_N \).
5. Use Eq. (3) to determine \( q_i \) for each surface of known \( T_i \) and \( T_i \) for each surface of known \( q_i \).
   - Treatment of the virtual surface corresponding to an opening (aperture) of area \( A_i \), through which the interior surfaces of an enclosure exchange radiation with large surroundings at \( T_{sur} \):
6. Approximate the opening as blackbody of area \( A_i \), temperature \( T_i = T_{sur} \), and properties, \( \epsilon_i = \alpha_i = 1 \).
Two-Surface Enclosure

- Simplest enclosure for which radiation exchange is exclusively between two surfaces and a single expression for the rate of radiation transfer may be inferred from a network representation of the exchange.

\[ q_1 = -q_2 = q_{12} = \frac{\sigma(T_1^4 - T_2^4)}{1 - \varepsilon_1 A_1 + \frac{1}{A_1 F_{12}} + \frac{1 - \varepsilon_2}{\varepsilon_2 A_2}} \]

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Radiation Shields

- High reflectivity (low \( \alpha = \varepsilon \)) surface(s) inserted between two surfaces for which a reduction in radiation exchange is desired.
- Consider use of a single shield in a two-surface enclosure, such as that associated with large parallel plates:

Note that, although rarely the case, emissivities may differ for opposite surfaces of the shield.
The Reradiating Surface

- An idealization for which $G_R = J_R$. Hence $q_R = 0$ and $J_R = E_{hr}$.
- Approximated by surfaces that are well insulated on one side and for which convection is negligible on the opposite (radiating) side.
- Three-Surface Enclosure with a Reradiating Surface:

\[
q_1 = -q_2 = \frac{\sigma (T_1^4 - T_2^4)}{1 - \varepsilon_1} + \frac{1}{\varepsilon_1 A_1 F_{12}} + \left[ \frac{1}{\varepsilon_2 A_2 F_{12}} + \frac{1}{\varepsilon_2 A_2 F_{1R}} \right] + \frac{1}{\varepsilon_2 A_2 F_{1R}}
\]

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Continue

- Temperature of reradiating surface $T_R$ may be determined from knowledge of its radiosity $J_R$. With $q_R = 0$, a radiation balance on the surface yields

\[
\frac{J_{1R} - J_{2R}}{(1/A_1 F_{1R})} = \frac{J_{R} - J_{2R}}{(1/A_2 F_{2R})}
\]

\[
T_R = \left(\frac{J_R}{\sigma}\right)^{1/4}
\]

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Multimode Effects

- In an enclosure with conduction and convection heat transfer to or from one or more surfaces, the foregoing treatments of radiation exchange may be combined with surface energy balances to determine thermal conditions.
- Consider a general surface condition for which there is external heat addition (e.g., electrically), as well as conduction, convection and radiation.

\[ q_{i, ext} = q_{i, rad} + q_{i, conv} + q_{i, rad} \]

\( q_{i, rad} \rightarrow \) Appropriate analysis for \( N \)-surface, two-surface, etc., enclosure.

Problem: Furnace in Spacecraft Environment

Problem 13.88: Power requirement for a cylindrical furnace with two reradiating surfaces and an opening to large surroundings.

**KNOWN:** Cylindrical furnace of diameter \( D = 90 \text{ mm} \) and overall length \( L = 180 \text{ mm} \). Heating elements maintain the refractory lining (\( e = 0.8 \)) of section (1), \( L_1 = 135 \text{ mm} \), at \( T_1 = 800^\circ \text{C} \). The bottom (2) and upper (3) sections are refractory lined, but are insulated. Furnace operates in a spacecraft vacuum environment.

**FIND:** Power required to maintain the furnace operating conditions with the surroundings at \( 23^\circ \text{C} \).
ASSUMPTIONS: (1) All surfaces are diffuse gray, and (2) Uniform radiosity over the sections 1, 2, and 3.

ANALYSIS: By defining the furnace opening as the hypothetical area $A_4$, the furnace can be represented as a four-surface enclosure.

The power required to maintain $A_1$ at $T_1$ is $q_1$, the net radiation leaving $A_1$.

To obtain $q_1$, we must determine the radiosity at each surface by simultaneously solving radiation energy balance equations of the form

$$ q_i = \frac{E_{bi}}{(1 - \varepsilon_i) / \varepsilon_i A_i} - \sum_{j=1}^{N} \frac{J_j - J_i}{1/A_i F_{ij}} \quad (1,2) $$

However, since $\varepsilon_4 = 1$, $J_4 = E_{34}$, and only three energy balances are needed for $A_1$, $A_3$, and $A_3$.

$$ A_1: \frac{E_{b1} - J_1}{(1 - \varepsilon_1) / \varepsilon_1 A_1} = \frac{J_1 - J_2}{1/A_1 F_{12}} + \frac{J_1 - J_3}{1/A_1 F_{13}} + \frac{J_1 - J_4}{1/A_1 F_{14}} \quad (3) $$

$$ A_2: 0 = \frac{J_2 - J_1}{1/A_2 F_{21}} + \frac{J_2 - J_3}{1/A_2 F_{23}} + \frac{J_2 - J_4}{1/A_2 F_{24}} \quad (4) $$

$$ A_3: 0 = \frac{J_3 - J_1}{1/A_3 F_{31}} + \frac{J_3 - J_2}{1/A_3 F_{32}} + \frac{J_3 - J_4}{1/A_3 F_{34}} \quad (5) $$

where $q_2 = q_3 = 0$ since the surfaces are insulated (adiabatic) and hence reradiating.

From knowledge of $J_1$, $q_1$ can be determined using Eq. (1).

Of the $N^2 = 4^2 = 16$ view factors, $N(N - 1)/2 = 6$ must be independently evaluated, while the remaining can be determined by the summation rule and appropriate reciprocity relations. The six independently determined $F_{ij}$ are:

By inspection: (1) $F_{22} = 0$  
(2) $F_{44} = 0$
Problem: Furnace in Spacecraft Environment (cont)

Coaxial parallel disks: From Table 13.2,

\[ F_{24} = 0.5 \left\{ S - \left[ S^2 - 4 \left( \frac{r_1}{r_2} \right)^2 \right]^{1/2} \right\} = 0.05573 \]

where

\[ S = 1 + \frac{1 + R_4^2}{R_2^2} = 1 + \frac{1 + 0.250^2}{0.250^2} = 18.00 \quad R_2 = r_2 / L = 45 / 180 = 0.250 \quad R_4 = r_4 / L = 0.250 \]

Enclosure 1-2-2*: From the summation rule for \( A_2 \),

\[ F_{21} = 1 - F_{22} = 1 - 0.9167 = 0.0903 \]

where \( F_{22} \) can be evaluated from the coaxial parallel disk relation, Table 13.2, with \( R_2 = r_2 / L_1 = 45 / 135 = 0.333 \), \( R_2' = r_2 / L_1 = 0.333 \), and \( S = 11.00 \).

From the summation rule for \( A_1 \),

\[ F_{12} = A_2 F_{21} / A_1 = \left[ \pi \left( \frac{0.090 m}{4} \right)^2 / 4 \right] \times 0.9083 / \pi \times 0.090 m \times 0.135 m = 0.1514 \]

Enclosure 2'-3-4: From the summation rule for \( A_4 \),

\[ F_{43} = 1 - F_{44} = 1 - 0.3820 = 0 = 0.6180 \]

where \( F_{44} = 0 \) and using the coaxial parallel disk relation from Table 13.2, \( F_{44'} = 0.3820 \) with \( R_4 = r_4 / L_2 = 45 / 45 = 1 \), \( R_4' = r_2 / L_2 = 1 \), and \( S = 3 \).

The View Factors: Using summation rules and appropriate reciprocity relations, the remaining 10 view factors can be evaluated. Written in matrix form, the \( F_{ij} \) are

\[
\begin{bmatrix}
0.6972* & 0.1514 & 0.09704 & 0.05438 \\
0.9083* & 0* & 0.03597 & 0.05573* \\
0.2911 & 0.01798 & 0.3819 & 0.3090 \\
0.3262 & 0.05573 & 0.6180* & 0* 
\end{bmatrix}
\]

The \( F_{ij} \) shown with an asterisk were independently determined.

From knowledge of the relevant view factors, the energy balances, Eqs. (3, 4, 5), can be solved simultaneously to obtain the radiossies,

\[ J_1 = 73,084 \text{ W/m}^2 \quad J_2 = 67,723 \text{ W/m}^2 \quad J_3 = 36,609 \text{ W/m}^2 \]

The net heat rate leaving \( A_1 \) can be evaluated using Eq. (1) written as

\[ q_1 = \frac{E_{14} - J_1}{(1 - \epsilon_1) / \epsilon_1 A_1} = \frac{(75,159 - 73,084) \text{ W/m}^2}{(1 - 0.8) / 0.8 \times 0.03817 \text{ m}^2} = 317 \text{ W} \]

where \( E_{14} = \sigma T_1^4 = \sigma(800 + 273)K^4 = 75,159 \text{ W/m}^2 \) and \( A_1 = \pi D L_1 = \pi \times 0.090 m \times 0.135 m = 0.03817 \text{ m}^2 \).

COMMENTS: Recognize the importance of defining the furnace opening as the hypothetical area \( A_4 \) which completes the four-surface enclosure representing the furnace. The temperature of \( A_4 \) is that of the surroundings and its emissivity is unity since it absorbs all radiation incident on it.
Problem 13.93: Assessment of ceiling radiative properties for an ice rink in terms of ability to maintain surface temperature above the dew point.

**KNOWN:** Ice rink with prescribed ice, rink air, wall, ceiling and outdoor air conditions.

**FIND:** (a) Temperature of the ceiling, $T_c$, for an emissivity of 0.05 (highly reflective panels) or 0.94 (painted panels); determine whether condensation will occur for either or both ceiling panel types if the relative humidity of the rink air is 70%, and (b) Calculate and plot the ceiling temperature as a function of ceiling insulation thickness for $0.1 \leq t \leq 1$ m; identify conditions for which condensation will occur on the ceiling.

**SCHEMATIC:**

**ASSUMPTIONS:** (1) Rink comprised of the ice, walls and ceiling approximates a three-surface, diffuse-gray enclosure, (2) Surfaces have uniform radiosities, (3) Ice surface and walls are black, (4) Panels are diffuse-gray, and (5) Thermal resistance for convection on the outdoor side of the ceiling is negligible compared to the conduction resistance of the ceiling insulation.
Problem 13.93 (cont)

PROPERTIES: Psychrometric chart (Atmospheric pressure; dry bulb temperature, $T_{db} = T_{w,i}$ = 15°C; relative humidity, RH = 70%): Dew point temperature, $T_{dp}$ = 9.4°C.

ANALYSIS: Applying an energy balance to the inner surface of the ceiling and treating all heat rates as energy outflows,

$$-q_o - q_{\text{conv},c} - q_{\text{rad},c} = 0 \quad (1)$$

where the rate equations for each process are

$$q_o = \left( T_c - T_{\text{w,oc}} \right) / R_{\text{cond}} \quad R_{\text{cond}} = t / k A_c \quad (2,3)$$

$$q_{\text{conv},c} = h_i A_c \left( T_c - T_{\text{w,i}} \right) \quad (4)$$

$$q_{\text{rad},c} = \varepsilon E_b \left( T_c \right) A_c - \alpha A_w F_{wc} E_b \left( T_w \right) - \alpha A_i F_{ic} E_b \left( T_i \right) \quad (5)$$

Since the ceiling panels are diffuse-gray, $\varepsilon = 0.8$.

From Table 13.2 for parallel, coaxial disks $F_{ic} = 0.672$.

From the summation rule applied to the ice (i) and the reciprocity rule,

$$F_{ic} + F_{iw} = 1 \quad F_{iw} = F_{cw} \text{ (symmetry)}$$

$$F_{cw} = 1 - F_{ic}$$

$$F_{wc} = \left( A_c / A_w \right) F_{cw} = \left( A_c / A_w \right) \left( 1 - F_{ic} \right) = 0.410$$

where $A_c = \pi D^2/4$ and $A_w = \pi DL$.

Using the foregoing energy balance, Eq. (1), and the rate equations, Eqs. (2-5), the ceiling temperature is calculated using radiative properties for the two panel types,

<table>
<thead>
<tr>
<th>Ceiling panel</th>
<th>$\varepsilon$</th>
<th>$T_c$ (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reflective</td>
<td>0.05</td>
<td>14.0</td>
</tr>
<tr>
<td>Paint</td>
<td>0.94</td>
<td>8.6</td>
</tr>
</tbody>
</table>

Condensation will occur on the painted panel since $T_c < T_{dp}$.

(b) Applying the foregoing model for $0.1 \leq t \leq 1.0$ m, the following result is obtained
For the reflective panel ($\varepsilon = 0.05$), the ceiling surface temperature is considerably above the dew point. Therefore, condensation will not occur for the range of insulation thicknesses. For the painted panel ($\varepsilon = 0.94$), the ceiling surface temperature is always below the dew point, and condensation occurs for the range of insulation thicknesses.

**COMMENTS:** From the analysis, recognize that radiative exchange between the ice and the ceiling has the dominant effect on the ceiling temperature. With the reflective panel, the rate is reduced nearly 20-fold relative to that for the painted panel. With the painted panel ceiling, condensation will occur for most of the conditions likely to exist in the rink.