Problem 2.46 Thermal response of a plane wall to convection heat transfer.

- **KNOWN:** Plane wall, initially at a uniform temperature, is suddenly exposed to convective heating.
- **FIND:** (a) Differential equation and initial and boundary conditions which may be used to find the temperature distribution, $T(x,t)$; (b) Sketch $T(x,t)$ for the following conditions: initial ($t \leq 0$), steady-state ($t \rightarrow \infty$), and two intermediate times; (c) Sketch heat fluxes as a function of time at the two surfaces; (d) Expression for total energy transferred to wall per unit volume ($J/m^3$).
Solution

- ASSUMPTIONS: (1) One-dimensional conduction, (2) Constant properties, (3) No internal heat generation.

ANALYSIS: (a) For one-dimensional conduction with constant properties, the heat equation has the form,

\[
\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha} \frac{\partial T}{\partial t}
\]

and the conditions are:

\[
\begin{align*}
\text{Initial, } t \leq 0: & \quad T(x,0) = T_i \\
\text{Boundaries: } & \quad x = 0 \quad \frac{\partial T}{\partial x}(0) = 0 \quad \text{adiabatic} \\
& \quad x = L \quad -k \frac{\partial T}{\partial x}(L) = h[T(L,t) - T_\infty] \quad \text{convection}
\end{align*}
\]

Solution (cont.)

(b) The temperature distributions are shown on the sketch.

Note that the gradient at \(x = 0\) is always zero, since this boundary is adiabatic. Note also that the gradient at \(x = L\) decreases with time.

(c) The heat flux, \(q_x''(x,t)\), as a function of time, is shown on the sketch for the surfaces \(x = 0\) and \(x = L\).
Solution (cont.)

For the surface at $x = 0$, $q_x^o(0, t) = 0$ since it is adiabatic. At $x = L$ and $t = 0$, $q_x^o(L, 0)$ is a maximum

$$q_x^o(L, 0) = h[T(L, 0) - T_\infty]$$

where $T(L, 0) = T_i$. The gradient, and hence the flux, decrease with time.

(d) The total energy transferred to the wall may be expressed as

$$E_{in} = \int_0^\infty q_{conv}^o \, dx \, dt$$

$$E_{in} = hA_s \int_0^\infty (T_\infty - T(L, t)) \, dx \, dt$$

Dividing both sides by $A_s L$, the energy transferred per unit volume is

$$\frac{E_{in}}{V} = \frac{h}{L} \int_0^\infty [T_\infty - T(L, t)] \, dx \, dt \quad \left[ \text{J/m}^3 \right]$$

Problem 2.28 Surface heat fluxes, heat generation and total rate of radiation absorption in an irradiated semi-transparent material with a prescribed temperature distribution.

The steady-state temperature distribution in a semitransparent material of thermal conductivity $k$ and thickness $L$ exposed to laser irradiation is of the form $T(x)$, where $A$, $a$, $B$, and $C$ are known constants. For this situation, radiation absorption in the material is manifested by a distributed heat generation term, $q(x)$.
Solution

KNOWN: Temperature distribution in a semi-transparent medium subjected to radiative flux.

FIND: (a) Expressions for the heat flux at the front and rear surfaces, (b) Heat generation rate $q(x)$, (c) Expression for absorbed radiation per unit surface area in terms of $A$, $a$, $B$, $C$, $L$, and $k$.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in medium, (3) Constant properties, (4) All laser irradiation is absorbed and can be characterized by an internal volumetric heat generation term $q(x)$.

Solution (cont.)

ANALYSIS: (a) Knowing the temperature distribution, the surface heat fluxes are found using Fourier’s law,

$$ q_x''(0) = -k \left[ \frac{dT}{dx} \right] = -k \left[ \frac{A}{ka^2} (-a) e^{-ax} + B \right] $$

*Front Surface, $x=0$:*

$$ q_x''(0) = -k \left[ \frac{A}{ka} \cdot 1 + B \right] = \left[ \frac{A}{a} + kB \right] $$

*Rear Surface, $x=L$:*

$$ q_x''(L) = -k \left[ \frac{A}{ka} e^{-aL} + B \right] = \left[ \frac{A}{a} e^{-aL} + kB \right] $$

(b) The heat diffusion equation for the medium is

$$ \frac{d}{dx} \left( \frac{dT}{dx} \right) + \frac{q}{k} = 0 \quad \text{or} \quad q = -k \frac{d}{dx} \left( \frac{dT}{dx} \right) $$

$$ q(x) = -k \frac{d}{dx} \left[ \frac{A}{ka} e^{-ax} + B \right] = Ae^{-ax}. $$
Problem 3.23: Assessment of thermal barrier coating (TBC) for protection of turbine blades. Determine maximum blade temperature with and without TBC.

- Consider a tube wall of inner and outer radii \( r_i \) and \( r_o \), whose temperatures are maintained at \( T_i \) and \( T_o \) respectively. The thermal conductivity of the cylinder is temperature dependent and may be represented by an expression of the form \( k = k_0(1 + aT) \), where \( k_0 \) and \( a \) are constants. Obtain an expression for the heat transfer per unit length of the tube. What is the thermal resistance of the tube wall?
**Solution**

- **Schematic:**

![Schematic Diagram](image)

- **Assumption:**
  1. One-dimensional, steady-state conduction in a composite plane wall,
  2. Constant properties,

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**Solution (cont.)**

**ANALYSIS:** For a unit area, the total thermal resistance with the TBC is

\[ R_{\text{tot},w} = h_o^{-1} + \left( \frac{1}{k} \right)_{zz} + R_{Tc}^{-1} + \left( \frac{1}{k} \right)_{ln} + h_i^{-1} \]

\[ R_{\text{tot},w} = \left( 10^{-3} + 3.85 \times 10^{-4} + 4 \times 10^{-4} + 2 \times 10^{-4} + 3.5 \times 10^{-3} \right) \text{m}^2 \cdot \text{K}/\text{W} = 3.69 \times 10^{-3} \text{m}^2 \cdot \text{K}/\text{W} \]

With a heat flux of

\[ q_{\text{w}}^* = \frac{T_{\text{w},0} - T_{\text{w},i}}{R_{\text{tot},w}} = \frac{1300 \text{ K}}{3.69 \times 10^{-3} \text{ m}^2 \cdot \text{K}/\text{W}} = 3.52 \times 10^5 \text{ W/m}^2 \]

the inner and outer surface temperatures of the Inconel are

\[ T_{\text{b},i(w)} = T_{\text{w},i} + \left( q_{\text{w}}^*/h_i \right) = 400 \text{ K} + \left( 3.52 \times 10^5 \text{ W/m}^2 / 500 \text{ W/m}^2 \cdot \text{K} \right) = 1104 \text{ K} \]

\[ T_{\text{b},o(w)} = T_{\text{w},o} + \left[ \left( 1/h_i \right) + \left( 1/k \right)_{ln} \right] q_{\text{w}}^* = 400 \text{ K} + \left( 2 \times 10^{-3} + 2 \times 10^{-4} \right) \text{m}^2 \cdot \text{K}/\text{W} \left( 3.52 \times 10^5 \text{ W/m}^2 \right) = 1174 \text{ K} \]

Without the TBC, \( R_{\text{tot},wo} = h_o^{-1} + \left( 1/k \right)_{zz} + h_i^{-1} = 3.2 \times 10^{-3} \text{ m}^2 \cdot \text{K}/\text{W} \), and \( q_{\text{w},o}^* = \left( T_{\text{w},o} - T_{\text{w},i} \right)/R_{\text{tot},wo} = \left( 1300 \text{ K} / 3.2 \times 10^{-3} \text{ m}^2 \cdot \text{K}/\text{W} \right) = 4.06 \times 10^5 \text{ W/m}^2 \). The inner and outer surface temperatures of the Inconel are then

\[ T_{\text{b},i(wo)} = T_{\text{w},i} + \left( q_{\text{w},o}^*/h_i \right) = 400 \text{ K} + \left( 4.06 \times 10^5 \text{ W/m}^2 / 500 \text{ W/m}^2 \cdot \text{K} \right) = 1212 \text{ K} \]

\[ T_{\text{b},o(wo)} = T_{\text{w},o} + \left[ \left( 1/h_i \right) + \left( 1/k \right)_{ln} \right] q_{\text{w},o}^* = 400 \text{ K} + \left( 2 \times 10^{-3} + 2 \times 10^{-4} \right) \text{m}^2 \cdot \text{K}/\text{W} \left( 4.06 \times 10^5 \text{ W/m}^2 \right) = 1293 \text{ K} \]
Problem 3.62: Suitability of a composite spherical shell for storing radioactive wastes in oceanic waters.

- A composite spherical shell of inner radius $r_1 = 0.25 \text{ m}$ is constructed from lead of outer radius $r_2 = 0.30 \text{ m}$ and AISI 302 stainless steel of outer radius $r_3 = 0.31 \text{ m}$. The cavity is filled with radioactive wastes that generate heat at a rate of $q = 5 \times 10^5 \text{ W/m}^3$. It is proposed to submerge the container in oceanic waters that are at a temperature of $T_\infty = 10^\circ \text{C}$ and provide a uniform convection coefficient of $h = 500 \text{ W/m}^2\cdot\text{K}$ at the outer surface of the container. Are there any problems associated with this proposal?

FIND: Inner surface temperature, $T_1$, of lead (proposal is flawed if this temperature exceeds the melting point).

Solution

- Schematic:

- Assumption:
  1. One-dimensional conduction,
  2. Steady-state conditions,
  3. Constant properties at 300K,
  4. Negligible contact resistance.
Problem 3.91  Thermal conditions in a gas-cooled nuclear reactor with a tubular thorium fuel rod and a concentric graphite sheath:  (a) Assessment of thermal integrity for a generation rate of $q$. (b) Evaluation of temperature distributions in the thorium and graphite for generation rates in the range $10^7 \leq q \leq 5 \times 10^8$.

A high-temperature, gas-cooled nuclear reactor consists of a composite cylindrical wall for which a thorium fuel element ($k \approx 57$ W/m·K) is encased in graphite ($k \approx 3$ W/m·K) and gaseous helium flows through an annular coolant channel. Consider conditions for which the helium temperature is $T_\infty = 600$ K and the convection coefficient at the outer surface of the graphite is $h = 2000$ W/m$^2$·K.
Solution

- Schematic:

- Assumption:
  (1) Steady-state conditions,  (2) One-dimensional conduction,  (3) Constant properties,  (4) Negligible contact resistance,  (5) Negligible radiation,  (6) Adiabatic surface at $r_1$

**PROPERTIES:**  *Table A.1,* Thorium:  $T_{\text{mp}} = 2000 $ K;  *Table A.2,* Graphite:  $T_{\text{mp}} \approx 2300 $ K.

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Solution (cont.)

**ANALYSIS:**  (a) The outer surface temperature of the fuel, $T_2$, may be determined from the rate equation

$$ q' = \frac{T_2 - T_{\infty}}{R'_{\text{tot}}} $$

where

$$ R'_{\text{tot}} = \frac{\ln \left( \frac{r_1}{r_2} \right)}{2\pi k_\ell} + \frac{1}{2\pi h} = \frac{\ln \left( 14/11 \right)}{2\pi (3 \text{ W/m} \cdot \text{K})} + \frac{1}{2\pi (0.014 \text{ m}) \left( \frac{2000 \text{ W/m}^2 \cdot \text{K}}{2 \times 57 \text{ W/m} \cdot \text{K}} \right)} = 0.0185 \text{ m} \cdot \text{K/W} $$

and the heat rate per unit length may be determined by applying an energy balance to a control surface about the fuel element. Since the interior surface of the element is essentially adiabatic, it follows that

$$ q' = q \left( r_2^2 - r_1^2 \right) = 10^8 \text{ W/m}^2 \times \pi \left( 0.011^2 - 0.008^2 \right) \text{ m}^2 = 17,907 \text{ W/m} $$

Hence,

$$ T_2 = qR'_{\text{tot}} + T_{\infty} = 17,907 \text{ W/m} \left( 0.0185 \text{ m} \cdot \text{K/W} \right) + 600 \text{ K} = 931 \text{ K} $$

With zero heat flux at the inner surface of the fuel element, Eq. C.14 yields

$$ T_1 = T_2 + \frac{4q'\ell}{4k_\ell} \left( 1 - \frac{r_1^2}{r_2^2} \right) \frac{q'\ell}{2k_\ell} \ln \left( \frac{r_2}{r_1} \right) $$

$$ T_1 = 931 \text{ K} + \frac{10^8 \text{ W/m}^3 (0.011 \text{ m})^2}{4 \times 57 \text{ W/m} \cdot \text{K}} \left[ 1 - \left( \frac{0.008}{0.011} \right)^2 \right] - \frac{10^8 \text{ W/m}^3 (0.008 \text{ m})^2}{2 \times 57 \text{ W/m} \cdot \text{K}} \ln \left( \frac{0.011}{0.008} \right) $$

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$$ T_1 = 931 \text{ K} + 25 \text{ K} - 18 \text{ K} = 938 \text{ K} $$
Problem 5.12: Charging a thermal energy storage system consisting of a packed bed of aluminum spheres.

- Thermal energy storage systems commonly involve a packed bed of solid spheres, through which a hot gas flows if the system is being charged, or a cold gas if it is being discharged. In a charging process, heat transfer from the hot gas increases thermal energy stored within the colder spheres; during discharge, the stored energy decreases as heat is transferred from the warmer spheres to the cooler gas.

![Schematic](image)

Solution

- Schematic:

![Schematic](image)

- Assumption:
  1. Negligible heat transfer to or from a sphere by radiation or conduction due to contact with other spheres,
  2. Constant properties.
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Problem 5.16: Heating of coated furnace wall during start-up.

FIND: (a) Time required for surface of wall to reach a prescribed temperature, (b) Corresponding value of film surface temperature.

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Solution

ANALYSIS: The overall coefficient for heat transfer from the surface of the steel to the gas is

\[ U = (R_f^e)^{-1} = \left( \frac{1}{h} + R_f^e \right)^{-1} = \left( \frac{1}{25 \text{ W/m}^2 \cdot \text{K}} + 10^{-2} \text{ m}^2 \cdot \text{K/W} \right)^{-1} = 20 \text{ W/m}^2 \cdot \text{K} \]

Hence,

\[ \text{Bi} = \frac{UL}{k} = \frac{20 \text{ W/m}^2 \cdot \text{K} \times 0.01 \text{ m}}{60 \text{ W/m} \cdot \text{K}} = 0.0033 \]

and the lumped capacitance method can be used.

(a) It follows that

\[ \frac{T - T_\infty}{T_1 - T_\infty} = \exp(-t/\tau_1) = \exp(-t/RC) = \exp(-Ut/\rho L c) \]

\[ t = -\frac{\rho L c}{U} \ln \frac{T - T_\infty}{T_1 - T_\infty} = \frac{7850 \text{ kg/m}^3 (0.01 \text{ m}) 430 \text{ J/kg} \cdot \text{K}}{20 \text{ W/m}^2 \cdot \text{K}} \ln \frac{1200 - 1300}{300 - 1300} \]

\[ t = 3886 \text{s} = 1.08 \text{h} \]

(b) Performing an energy balance at the outer surface (s,0),

\[ h(T_{so} - T_{so}) = (T_{s,0} - T_{s,i})/R_f^e \]

\[ T_{s,0} = \frac{h T_{so} + T_{s,i} / R_f^e}{h + (1/R_f^e)} = \frac{25 \text{ W/m}^2 \cdot \text{K} \times 1300 \text{ K} + 1200 \text{ K} / 10^{-2} \text{ m}^2 \cdot \text{K/W}}{(25 + 100) \text{ W/m}^2 \cdot \text{K}} \]

\[ T_{s,0} = 1220 \text{ K} \]