SOLVED PROBLEMS

The First Law of Thermodynamics (Closed System)

Problem 1

Determine the volume change when 10 kg of saturated water is completely vaporized at a pressure of (a) 1 kPa, (b) 260 kPa, and (c) 10 000 kPa.

Solution

Table C.2 provides the necessary values. The quantity being sought is $\Delta V = mv_{fg}$ where $v_{fg} = v_g - v_f$. Note that P is given in MPa.

(a) 1 kPa = 0.001 MPa. Thus, v_{fg} = 129.2 – 0.001 = 129.2 m^3/kg.

$$\therefore \Delta V = 1292 \text{ m}^3$$

(b) At 0.26 MPa we must interpolate[2] if we use the tables. The tabulated values are used at 0.2 MPa and 0.3 MPa:

$$v_g = \frac{0.26-0.2}{0.3-0.2}(0.6058-0.8857)+0.8857 = 0.718 \text{ m}^3/\text{kg}$$

The value for v_f is, to four decimal places, 0.0011 m^3/kg at 0.2 MPa and at 0.3 MPa; hence, no need to interpolate for v_f. We then have

$$v_{fg} = 0.718-0.0011 = 0.717 \text{ m}^3/\text{kg}. \qquad \therefore \Delta V = 7.17 \text{ m}^3$$

(c) At 10 MPa, v_{fg} = 0.01803 – 0.00145 = 0.01658 m^3/kg so that

$$\Delta V = 0.1658 \text{ m}^3$$

Problem 2

Two kilograms of water are contained in a constant-pressure cylinder held at 2.2 MPa. Heat is added until the temperature reaches 800°C. Determine the final volume of the container.

Solution
Use Table C.3. Since 2.2 MPa lies between 2 MPa and 2.5 MPa, the specific volume is interpolated to be

$$v = 0.2467 + 0.4(0.1972 - 0.2467) = 0.227 \text{ m}^3/\text{kg}$$

The final volume is then

$$V = mv = 2 \times 0.227 = 0.454 \text{ m}^3$$

The linear interpolation above results in a less accurate number than the numbers in the table. So, the final number has fewer significant digits.

Problem 3

An automobile tire with a volume of 0.6 m^3 is inflated to a gage pressure of 200 kPa. Calculate the mass of air in the tire if the temperature is 20°C using the ideal-gas equation of state.

Solution
Air is assumed to be an ideal gas at the conditions of this example. In the ideal-gas equation, $PV = mRT$, we use absolute pressure and absolute temperature. Thus, using P_{atm} = 100 kPa (to use a pressure of 101 kPa is unnecessary; the difference of 1 percent is not significant in most engineering problems):

$$P_{abs} = P_{gage} + P_{atm} = 200 + 100 = 300 \text{ kPa} \quad \text{and} \quad T = T_C + 273 = 20 + 273 = 293 \text{ K}$$

The mass is then calculated to be

$$m = \frac{PV}{RT} = \frac{(300\,000 \text{ N/m}^2)(0.6 \text{ m}^3)}{(287 \text{ N}\cdot\text{m/kg}\cdot\text{K})(293 \text{ K})} = 2.14 \text{ kg}$$

Problem 4

A piston–cylinder assembly contains 0.1 kg wet steam of quality 0.7 at 1 bar. If 200 kJ energy is transferred as heat at constant pressure, determine the final state of the steam and the work done by the steam.

Solution :

We read the following values from the steam tables (see Appendix – 1):

$P = 1$ bar : $t = 99.632$°C ; $v_f = 0.001\ 043\ 4$ m³/kg ; $v_g = 1.694$ m³/kg

$$h_f = 417.54 \text{ kJ/kg}; \; h_g = 2675.4 \text{ kJ/kg}$$

$$v_1 = Xv_g + (1 - X)v_f = 0.7 \times 1.694 + 0.3 \times 0.001\ 043\ 4 = 1.1861 \text{ m}^3\text{/kg}$$

$$h_1 = Xh_g + (1 - X)h_f = 0.7 \times 2675.4 + 0.3 \times 417.54 = 2123.304 \text{ kJ/kg}$$

Problem 4 (Cont.)

We know that for a constant pressure process $q = h_2 - h_1$. Therefore

$$h_2 = h_1 + q = 2123.304 + 10 \times 200 = 4123.304 \text{ kJ/kg}$$

Energy transferred is 200 kJ for 0.1 kg or (10×200) kJ/kg. Since $h_2 > h_g$, the steam exists in the superheated state at 1 bar. From superheated steam tables (see Appendix – 3) we read the following values.

$P = 1$ bar :	$t = 700$°C	$t = 800$°C
v	4.4900 m³/kg	4.9520 m³/kg
h	3928.2 kJ/kg	4158.3 kJ/kg

The value $h_2 = 4123.304$ kJ/kg lies in between 700°C and 800°C. Therefore, we interpolate to determine the temperature of steam.

$$4123.304 = 3928.2 + \frac{(4158.3 - 3928.2)\Delta t}{800 - 700}$$

or $\Delta t = 84.79$ or $t = 700 + 84.79 = 784.79$°C

The process followed by the steam is shown in Fig.E 4.28.

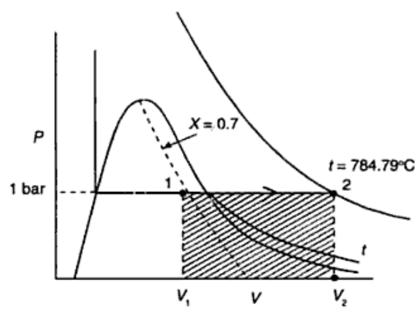


Fig.E 4.28. *P-V* diagram showing the path followed by steam. The shaded area represents the work done by steam.

Now we calculate the specific volume of steam at 784.79°C as

$$v_2 = 4.4900 + \frac{(4.9520 - 4.4900) \times 84.79}{800 - 700} = 4.8817 \text{ m}^3/\text{kg}$$

Problem 5

A 110-mm-diameter cylinder contains 100 cm³ of water at 60°C. A 50-kg piston sits on top of the water. If heat is added until the temperature is 200°C, find the work done.

Solution

The pressure in the cylinder is due to the weight of the piston and remains constant. Assuming a frictionless seal (this is always done unless information is given to the contrary), a force balance provides

$$mg = PA - P_{\text{atm}}A \qquad 50 \times 9.81 = (P - 100\,000)\frac{\pi \times 0.110^2}{4} \qquad \therefore P = 151\,600 \text{ Pa}$$

The atmospheric pressure is included so that absolute pressure results. The volume at the initial state 1 is given as

$$V_1 = 100 \times 10^{-6} = 10^{-4} \text{ m}^3$$

Using v_1 at 60°C, the mass is calculated to be

$$m = \frac{V_1}{v_1} = \frac{10^{-4}}{0.001017} = 0.09833 \text{ kg}$$

At state 2 the temperature is 200°C and the pressure is 0.15 MPa (this pressure is within 1 percent of the pressure of 0.1516 MPa, so it is acceptable). The volume is then

$$V_2 = mv_2 = 0.09833 \times 1.444 = 0.1420 \text{ m}^3$$

Finally, the work is calculated to be

$$W = P(V_2 - V_1) = 151.6(0.1420 - 0.0001) = 21.5 \text{ kJ}$$

Problem 6

Energy is added to a piston-cylinder arrangement, and the piston is withdrawn in such a way that the temperature (i.e., the quantity PV) remains constant. The initial pressure and volume are 200 kPa and 2 m³, respectively. If the final pressure is 100 kPa, calculate the work done by the ideal gas on the piston.

Solution

The work, using Eq. (3.4), can be expressed as

$$W_{1\text{-}2} = \int_{V_1}^{V_2} P dV = \int_{V_1}^{V_2} \frac{C}{V} dV$$

where we have used $PV = C$. To calculate the work we must find C and V_2. The constant C is found from

$$C = P_1 V_1 = 200 \times 2 = 400 \text{ kJ}$$

To find V_2 we use $P_2V_2 = P_1V_1$, which is, of course, the equation that would result from an isothermal process (constant temperature) involving an ideal gas. This can be written as

$$V_2 = \frac{P_1 V_1}{p_2} = \frac{200 \times 2}{100} = 4 \text{ m}^3$$

Finally,

$$W_{1\text{-}2} = \int_2^4 \frac{400}{V} dV = 400 \ln \frac{4}{2} = 277 \text{ kJ}$$

This is positive, since work is done during the expansion process by the gas.

Problem 7

The specific heat of superheated steam at approximately 150 kPa can be determined by the equation

$$C_p = 2.07 + \frac{T - 400}{1480} \text{ kJ/kg}\cdot°\text{C}$$

(a) What is the enthalpy change between 300 and 700°C for 3 kg of steam? Compare with the steam tables.

(b) What is the average value of C_p between 300 and 700°C based on the equation and based on the tabulated data?

Solution

(a) The enthalpy change is found to be

$$\Delta H = m\int_{T_1}^{T_2} C_p\,dT = 3\int_{300}^{700}\left(2.07 + \frac{T-400}{1480}\right)dT = 2565 \text{ kJ}$$

From the tables we find, using $P = 150$ kPa,

$$\Delta H = 3\times(3928 - 3073) = 2565 \text{ kJ}$$

(b) The average value $C_{p,\,\text{avg}}$ is found by using the relation

$$mC_{p,\,\text{avg}}\Delta T = m\int_{T_1}^{T_2} C_p\,dT \qquad \text{or} \qquad 3C_{p,\,\text{avg}}\times 400 = 3\int_{300}^{700}\left(2.07 + \frac{T-400}{1480}\right)dT$$

The integral was evaluated in part (a); hence, we have

$$C_{p,\,\text{avg}} = \frac{2565}{3\times 400} = 2.14 \text{ kJ/kg}\cdot°\text{C}$$

Using the values from the steam table, we have

$$C_{p,\,\text{avg}} = \frac{\Delta h}{\Delta T} = \frac{3928 - 3073}{400} = 2.14 \text{ kJ/kg}\cdot°\text{C}$$

Problem 8

Determine the heat transfer necessary to increase the pressure of 70 percent quality steam from 200 to 800 kPa, maintaining the volume constant at 2 m³. Assume a quasiequilibrium process.

Solution

For the constant-volume quasiequilibrium process the work is zero. The first law reduces to $Q = m(u_2 - u_1)$. The mass must be found. It is

$$m = \frac{V}{v_1} = \frac{2}{0.0011 + 0.7(0.8857 - 0.0011)} = 3.224 \text{ kg}$$

The internal energy at state 1 is

$$u_1 = 504.5 + 0.7(2529.5 - 504.5) = 1922 \text{ kJ/kg}$$

The constant-volume process demands that $v_2 = v_1 = 0.6203$ m³/kg. From the steam tables at 800 kPa we find, by interpolation, that

$$u_2 = \frac{0.6761 - 0.6203}{0.6761 - 0.6181}(3661 - 3853) = 3668 \text{ kJ/kg}$$

The heat transfer is then

$$Q = m(u_2 - u_1) = 3.224\times(3668 - 1922) = 5629 \text{ kJ}$$

Problem 9

Calculate the work necessary to compress air in an insulated cylinder from a volume of 2 m^3 to a volume of 0.2 m^3. The initial temperature and pressure are 20°C and 200 kPa, respectively.

Solution

We will assume that the compression process is approximated by a quasiequilibrium process, which is acceptable for most compression processes, and that the process is adiabatic due to the presence of the insulation (we usually assume an adiabatic process anyhow since heat transfer is assumed to be negligible). The first law is then written as

$$-W = m(u_2 - u_1) = mC_v(T_2 - T_1)$$

The mass is found from the ideal-gas equation to

$$m = \frac{PV}{RT} = \frac{200 \times 2}{0.287(20+273)} = 4.757 \text{ kg}$$

The final temperature T_2 is found for the adiabatic quasiequilibrium process from Eq. (4.40); it is

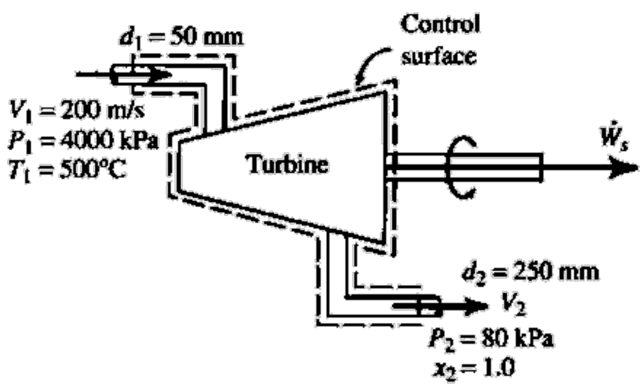
$$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{k-1} = 293\left(\frac{2}{0.2}\right)^{1.4-1} = 736 \text{ K}$$

Finally,

$$W = -4.757 \text{ kg}\left(0.717\ \frac{\text{kJ}}{\text{kg} \cdot \text{K}}\right)(736 - 293) \text{ K} = -1510 \text{ kJ}$$

Problem 10

Steam enters a turbine at 4000 kPa and 500°C and leaves as shown. For an inlet velocity of 200 m/s calculate the turbine power output. Neglect any heat transfer and kinetic energy change. Show that the kinetic energy change is negligible.



Problem 10 (Cont.)

Solution

The energy equation in the form of Eq. (4.55) is $-\dot{W}_T = \dot{m}(h_2 - h_1)$. We find $\dot{m}$ as follows:

$$\dot{m} = \rho_1 A_1 V_1 = \frac{1}{v_1} A_1 V_1 = \frac{1}{0.08643} \times \pi \times 0.025^2 \times 200 = 4.544 \text{ kg/s}$$

The enthalpies are found from Table C.3 to be

$$h_1 = 3445 \text{ kJ/kg} \qquad h_2 = 2666 \text{ kJ/kg}$$

The maximum power output is then

$$\dot{W}_T = -4.544 \times (2666 - 3445) = 3540 \text{ kJ/s} \quad \text{or} \quad 3.54 \text{ MW}$$

To show that the kinetic energy change is negligible we must calculate the exiting velocity:

$$V_2 = \frac{A_1 V_1 \rho_1}{A_2 \rho_2} = \frac{\pi \times 0.025^2 \times 200/0.08643}{\pi \times 0.125^2/2.087} = 193 \text{ m/s}$$

The kinetic energy change is then

$$\Delta KE = \dot{m}\left(\frac{V_2^2 - V_1^2}{2}\right) = 4.544 \times \frac{193^2 - 200^2}{2} = -6250 \text{ J/s} \quad \text{or} \quad -6.25 \text{ kJ/s}$$

Problem 11

Determine the maximum pressure increase provided by a 10-hp pump with a 6-cm-diameter inlet and a 10-cm-diameter outlet. The inlet velocity of the water is 10 m/s.

Solution

The energy equation (4.53) is used. By neglecting the heat transfer and assuming no increase in internal energy, we establish the maximum pressure rise. Neglecting the potential energy change, the energy equation takes the form

$$-\dot{W}_S = \dot{m}\left(\frac{P_2 - P_1}{\rho} + \frac{V_2^2 - V_1^2}{2}\right)$$

The velocity V_1 is given and V_2 is found from the continuity equation as follows, recognizing that $\rho_1 = \rho_2$,

$$A_1 V_1 = A_2 V_2 \qquad \pi \times 0.03^2 \times 10 = \pi \times 0.05^2 \times V_2 \qquad \therefore V_2 = 3.6 \text{ m/s}$$

The mass flow rate, needed in the energy equation, is then, using $\rho = 1000 \text{ kg/m}^3$,

$$\dot{m} = \rho A_1 V_1 = 1000 \times \pi \times 0.03^2 \times 10 = 28.27 \text{ kg/s}$$

Recognizing that the pump work is negative, the energy equation is

$$-(-10) \times 746 = 28.27\left[\frac{\Delta P}{1000} + \frac{3.6^2 - 10^2}{2 \times 9.81}\right]. \qquad \therefore \Delta P = 268\ 000 \text{ Pa}$$

Problem 12

4–134 Two rigid tanks are connected by a valve. Tank A contains 0.2 m^3 of water at 400 kPa and 80 percent quality. Tank B contains 0.5 m^3 of water at 200 kPa and 250°C. The valve is now opened, and the two tanks eventually come to the same state. Determine the pressure and the amount of heat transfer when the system reaches thermal equilibrium with the surroundings at 25°C. *Answers:* 3.17 kPa, 2170 kJ

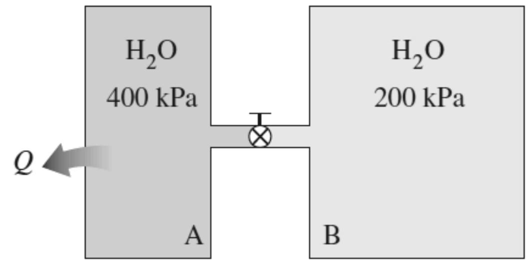


FIGURE P4–134

Problem 12 (Cont.)

$$\underbrace{E_{in} - E_{out}}_{\substack{\text{Net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{system}}_{\substack{\text{Change in internal, kinetic,}\\ \text{potential, etc. energies}}}$$

$$-Q_{out} = \Delta U = (\Delta U)_A + (\Delta U)_B \quad (\text{since } W = \text{KE} = \text{PE} = 0)$$

$$Q_{out} = -[U_{2,A+B} - U_{1,A} - U_{1,B}]$$

$$= -[m_{2,\text{total}} u_2 - (m_1 u_1)_A - (m_1 u_1)_B]$$

The properties of water in each tank are (Tables A-4 through A-6)

Tank A:

$$\left.\begin{array}{l} P_1 = 400 \text{ kPa} \\ x_1 = 0.80 \end{array}\right\} \begin{array}{ll} \boldsymbol{v}_f = 0.001084, & \boldsymbol{v}_g = 0.46242 \text{ m}^3/\text{kg} \\ u_f = 604.22, & u_{fg} = 1948.9 \text{ kJ/kg} \end{array}$$

$$\boldsymbol{v}_{1,A} = \boldsymbol{v}_f + x_1 \boldsymbol{v}_{fg} = 0.001084 + [0.8 \times (0.46242 - 0.001084)] = 0.37015 \text{ m}^3/\text{kg}$$

$$u_{1,A} = u_f + x_1 u_{fg} = 604.22 + (0.8 \times 1948.9) = 2163.3 \text{ kJ/kg}$$

Problem 12 (Cont.)

Tank B:

$$\left.\begin{array}{l} P_1 = 200 \text{ kPa} \\ T_1 = 250°\text{C} \end{array}\right\} \begin{array}{l} v_{1,B} = 1.1989 \text{ m}^3/\text{kg} \\ u_{1,B} = 2731.4 \text{ kJ/kg} \end{array}$$

$$m_{1,A} = \frac{V_A}{v_{1,A}} = \frac{0.2 \text{ m}^3}{0.37015 \text{ m}^3/\text{kg}} = 0.5403 \text{ kg}$$

$$m_{1,B} = \frac{V_B}{v_{1,B}} = \frac{0.5 \text{ m}^3}{1.1989 \text{ m}^3/\text{kg}} = 0.4170 \text{ kg}$$

$$m_t = m_{1,A} + m_{1,B} = 0.5403 + 0.4170 = 0.9573 \text{ kg}$$

$$v_2 = \frac{V_t}{m_t} = \frac{0.7 \text{ m}^3}{0.9573 \text{ kg}} = 0.73117 \text{ m}^3/\text{kg}$$

$$\left.\begin{array}{l} T_2 = 25°\text{C} \\ v_2 = 0.73117 \text{ m}^3/\text{kg} \end{array}\right\} \begin{array}{ll} v_f = 0.001003, & v_g = 43.340 \text{ m}^3/\text{kg} \\ u_f = 104.83, & u_{fg} = 2304.3 \text{ kJ/kg} \end{array}$$

Problem 12 (Cont.)

Thus at the final state the system will be a saturated liquid-vapor mixture since $v_f < v_2 < v_g$. Then the final pressure must be

$$P_2 = P_{\text{sat @ 25 °C}} = \mathbf{3.17\ kPa}$$

Also,

$$x_2 = \frac{v_2 - v_f}{v_{fg}} = \frac{0.73117 - 0.001}{43.340 - 0.001} = 0.01685$$

$$u_2 = u_f + x_2 u_{fg} = 104.83 + (0.01685 \times 2304.3) = 143.65 \text{ kJ/kg}$$

Substituting, $Q_{\text{out}} = -[(0.9573)(143.65) - (0.5403)(2163.3) - (0.4170)(2731.4)] = \mathbf{2170\ kJ}$